Higgs Boson Decays in the Minimal Supersymmetric Standard Model with Radiative Higgs Sector CP Violation

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Abstract

We re–evaluate the decays of the Higgs bosons in the minimal supersymmetric standard model (MSSM) where the tree-level CP invariance of the Higgs potential is explicitly broken by the loop effects of the third–generation squarks with CP–violating soft–breaking Yukawa interactions. This study is based on the mass matrix of the neutral Higgs bosons that is valid for arbitrary values of all the relevant MSSM parameters. It extends the previous work considerably by including neutral Higgs–boson decays into virtual gauge bosons and those into top–squark pairs, by implementing squark–loop contributions to the two–gluon decay channel, and by incorporating the decays of the charged Higgs boson. The constraints from the electron electric dipole moment on the CP phases are also discussed. We find that the branching fractions of both the neutral and charged Higgs–boson decays and their total decay widths depend strongly on the CP phases of the top (and bottom) squark sectors through the loop–induced neutral Higgs boson mixing as well as the direct couplings of the neutral Higgs bosons to top squark pairs.

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I. INTRODUCTION

The soft CP violating Yukawa interactions in the minimal supersymmetric standard model (MSSM) cause the CP–even and CP–odd neutral Higgs bosons to mix via loop corrections [1–5]. Although the mixing is a radiative effect, the induced CP violation in the MSSM Higgs sector can be large enough to affect the Higgs phenomenology significantly at present and future colliders [1,3,5–11].

In the light of the possible large CP–violating mixing, we have studied in Ref. [7] all the dominant two–body decay branching fractions of the three neutral Higgs bosons based on the mass matrix derived by Pilaftsis and Wagner [3]. The mass matrix, however, is not applicable for large squark mass splitting. In the present work, we re–evaluate all the two–body decay modes of the neutral Higgs bosons with the newly–calculated mass matrix [4] which is valid for any values of the soft–breaking parameters. Furthermore, we extend the work significantly by including the Higgs–boson decay modes containing virtual gauge bosons and those into squark pairs and, also by taking into account the squark–loop contributions to the gluon–gluon decay modes. In addition, we study the dominant decay modes of the charged Higgs boson in the presence of the non–trivial CP–violating mixing.

This paper is organized as follows. In Sec. II we give a brief review of the calculation [4] of the loop–induced CP–violating mass matrix of the three neutral Higgs bosons. We also consider the constraints by the electron electric dipole moment (EDM) on the space of the relevant supersymmetric parameters. In Sec. III we discuss the effects of the CP phases on the neutral Higgs boson decays. The effects of the CP phases on the charged Higgs boson decays are discussed in Sec. IV. Finally, we summarize our findings in Sec. V.

II. CP VIOLATION IN THE MSSM HIGGS SECTOR

The loop–corrected mass matrix of the neutral Higgs bosons in the MSSM can be calculated from the effective potential [12,13]

$$
\begin{aligned}
V_{\text{Higgs}} &= \tfrac{1}{2}m_1^2\left(\phi_1^2 + a_1^2\right) + \tfrac{1}{2}m_2^2\left(\phi_2^2 + a_2^2\right) - |m_{12}^2|\left(\phi_1\phi_2 - a_1a_2\right)\cos(\xi + \theta_{12}) \\
&+ |m_{12}^2|\left(\phi_1 a_2 + \phi_2 a_1\right)\sin(\xi + \theta_{12}) + \tfrac{\hat{g}^2}{8}\mathcal{D}^2 + \tfrac{1}{64\,\pi^2}\text{Str}\left[\mathcal{M}^4\left(\log\tfrac{\mathcal{M}^2}{Q^2} - \tfrac{3}{2}\right)\right],
\end{aligned} \tag{1}
$$

with $\mathcal{D} = \phi_2^2 + a_2^2 - \phi_1^2 - a_1^2$, $\hat{g}^2 = (g^2 + g'^2)/4$, and ϕ_i and a_i $(i = 1, 2)$ are the real fields of the neutral components of the two Higgs doublets:

$$
H_1^0 = \frac{1}{\sqrt{2}}\left(\phi_1 + ia_1\right), \qquad H_2^0 = \frac{e^{i\xi}}{\sqrt{2}}\left(\phi_2 + ia_2\right). \tag{2}
$$

The parameters g and g' are the $\text{SU}(2)_L$ and $\text{U}(1)_Y$ gauge couplings, respectively, and Q denotes the renormalization scale. All the tree–level parameters of the effective potential (1) such as m_1^2, m_2^2 and $m_{12}^2 = |m_{12}^2|\,e^{i\theta_{12}}$, are the running parameters evaluated at the scale Q. The potential (1) is then almost independent of Q up to two–loop–order corrections. The

super–trace is to be taken over all the bosons and fermions that couple to the Higgs fields.

The matrix $\mathcal{M}$ in Eq. (1) is the field–dependent mass matrix of all modes that couple to the Higgs bosons. The dominant contributions in the MSSM come from third generation quarks and squarks because of their large Yukawa couplings. The field–dependent masses of the third generation quarks are given by

$$m_b^2 = \frac{1}{2}|h_b|^2 \left(\phi_1^2 + a_1^2\right), \qquad m_t^2 = \frac{1}{2}|h_t|^2 \left(\phi_2^2 + a_2^2\right), \tag{3}$$

where h_b and h_t are the bottom and top Yukawa couplings, respectively. The corresponding squark mass matrices read:

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_t^2 - \frac{1}{8}\left(g^2 - \frac{g'^2}{3}\right)\mathcal{D} & -h_t^*\left[A_t^*\left(H_2^0\right)^* + \mu H_1^0\right] \\ -h_t\left[A_t H_2^0 + \mu^*\left(H_1^0\right)^*\right] & m_{\tilde{U}}^2 + m_t^2 - \frac{g'^2}{6}\mathcal{D} \end{pmatrix},$$

$$\mathcal{M}_{\tilde{b}}^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_b^2 + \frac{1}{8}\left(g^2 + \frac{g'^2}{3}\right)\mathcal{D} & -h_b^*\left[A_b^*\left(H_1^0\right)^* + \mu H_2^0\right] \\ -h_b\left[A_b H_1^0 + \mu^*\left(H_2^0\right)^*\right] & m_{\tilde{D}}^2 + m_b^2 + \frac{g'^2}{12}\mathcal{D} \end{pmatrix}. \tag{4}$$

where $m_{\tilde{Q}}^2$, $m_{\tilde{U}}^2$ and $m_{\tilde{D}}^2$ are the real soft SUSY–breaking squark-mass parameters, A_b and A_t are the complex soft SUSY–breaking trilinear parameters, and μ is the complex supersymmetric Higgsino mass parameter.

The mass matrix of the Higgs bosons (at vanishing external momenta) is then given by the second derivatives of the potential, evaluated at its minimum point

$$(\phi_1,\, \phi_2,\, a_1,\, a_2) = (\langle\phi_1\rangle,\, \langle\phi_2\rangle,\, \langle a_1\rangle,\, \langle a_2\rangle) = (v\cos\beta,\, v\sin\beta,\, 0,\, 0), \tag{5}$$

where $v = (\sqrt{2}\,G_F)^{-1/2} \simeq 246$ GeV. The massless state $G^0 = a_1\cos\beta - a_2\sin\beta$ is the would–be–Goldstone mode to be absorbed by the Z boson. We are thus left with a mass–squared matrix $\mathcal{M}_H^2$ for three physical states, $a\,(= a_1\sin\beta + a_2\cos\beta)$, ϕ_1 and ϕ_2. This matrix is real and symmetric, i.e. it has 6 independent entries. The diagonal entry for the pseudoscalar component a reads:

$$\mathcal{M}_H^2\Big|_{aa} = m_A^2 + \frac{3}{8\pi^2}\left\{\frac{|h_t|^2 m_t^2}{\sin^2\beta}g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\Delta_{\tilde{t}}^2 + \frac{|h_b|^2 m_b^2}{\cos^2\beta}g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)\Delta_{\tilde{b}}^2\right\}, \tag{6}$$

where m_A is the loop–corrected pseudoscalar mass in the CP invariant theories. The CP–violating entries of the mass matrix, which mix a with ϕ_1 and ϕ_2, are given by

$$\mathcal{M}_H^2\Big|_{a\phi_1} = \frac{3}{16\pi^2}\left\{\frac{m_t^2\Delta_{\tilde{t}}}{\sin\beta}\left[g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\left(X_t\cot\beta - 2\,|h_t|^2\,R_t\right) - \hat{g}^2\cot\beta\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right]\right. \tag{7}$$

$$\left. + \frac{m_b^2\Delta_{\tilde{b}}}{\cos\beta}\left[-g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)\left(X_b + 2\,|h_b|^2\,R_b'\right) + \left(\hat{g}^2 - 2\,|h_b|^2\right)\log\frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2}\right]\right\},$$

$$\mathcal{M}_H^2\Big|_{a\phi_2} = \frac{3}{16\pi^2}\left\{\frac{m_t^2\Delta_{\tilde{t}}}{\sin\beta}\left[-g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\left(X_t + 2\,|h_t|^2\,R_t'\right) + \left(\hat{g}^2 - 2\,|h_t|^2\right)\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right]\right. \tag{8}$$

$$\left. + \frac{m_b^2\Delta_{\tilde{b}}}{\cos\beta}\left[g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)\left(X_b\tan\beta - 2\,|h_b|^2\,R_b\right) - \hat{g}^2\tan\beta\log\frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2}\right]\right\}.$$

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where $g(x,y) = 2 - [(x+y)/(x-y)]\log(x/y)$. The size of these CP–violating entries is determined by the re–phasing invariant quantities

$$\Delta_{\tilde{t}} = \frac{\Im m(A_t \mu \mathrm{e}^{i\xi})}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}, \qquad \Delta_{\tilde{b}} = \frac{\Im m(A_b \mu \mathrm{e}^{i\xi})}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2}, \tag{9}$$

which measure the amount of CP violation in the top and bottom squark–mass matrices. In the CP–conserving limit, both $\Delta_{\tilde{t}}$ and $\Delta_{\tilde{b}}$ vanish, leading to $|m_{12}^2|\sin(\xi + \theta_{12}) = 0$. The definition of the mass–squared m_A^2 and the dimensionless quantities $X_{t,b}$, $R_{t,b}$ and $R'_{t,b}$, as well as the other CP–preserving entries of the mass matrix squared $\mathcal{M}_H^2$, can be found in Ref. [4]. The real and symmetric matrix $\mathcal{M}_H^2$ can now be diagonalized with an orthogonal matrix O;

$$\begin{pmatrix} a \\ \phi_1 \\ \phi_2 \end{pmatrix} = O \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}. \tag{10}$$

Our convention for the three mass eigenvalues is $m_{H_1} \leq m_{H_2} \leq m_{H_3}$.

The loop–corrected neutral–Higgs–boson sector depends on various parameters from the other sectors of the MSSM; m_A, $\tan\beta$, μ, A_t, A_b, the renormalization scale Q, and the real soft–breaking masses, $m_{\tilde{Q}}$, $m_{\tilde{U}}$, and $m_{\tilde{D}}$, as well as on the complex gluino–mass parameter $M_{\tilde{g}}$ through one–loop corrections to the top and bottom quark masses [14]. Noting that the size of the radiative Higgs sector CP violation is determined by the rephasing invariant combinations $A_t \mu \mathrm{e}^{i\xi}$ and $A_b \mu \mathrm{e}^{i\xi}$, see Eq. (9), we take for our numerical analysis the following set of parameters:

$$\begin{aligned} |A_t| = |A_b| &= 1 \text{ TeV}, & |\mu| &= 2 \text{ TeV}, \\ M_{\tilde{Q},\tilde{U},\tilde{D}} = |M_{\tilde{g}}| &= 0.5 \text{ TeV}, & \xi + \mathrm{Arg}(\mu) &= \mathrm{Arg}(M_{\tilde{g}}) = 0, \end{aligned} \tag{11}$$

under the constraint;

$$\Phi \equiv \mathrm{Arg}(A_t \mu \mathrm{e}^{i\xi}) = \mathrm{Arg}(A_b \mu \mathrm{e}^{i\xi}). \tag{12}$$

We vary the common phase Φ as well as m_A and $\tan\beta$ in the following numerical studies. Our choice of relatively large magnitudes of $|A_t\mu| = |A_b\mu|$ enhances CP–violation effects in the MSSM Higgs sector [1].

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[1] Our choice of the set of parameters satisfies the necessary condition to avoid a color and electric–charge breaking (CCB) minimum in the direction $|\tilde{Q}| = |\tilde{U}| = |H_2^0|$ [15]: $|A_t|^2 \leq 3(M_{\tilde{Q}}^2 + M_{\tilde{U}}^2 + m_2^2)$. But, according to the more general study [16], our relatively large values of $|A_{t,b}|$ and $|\mu|$ compared to those of $M_{\tilde{Q},\tilde{U},\tilde{D}}$ could give rise to a dangerous CCB minima in the potential which could be deeper than the electro–weak vacuum. Therefore, a detailed study of the CCB minima in the presence of the non–trivial CP–violating mixing among the neutral Higgs bosons deserves further analysis.

Before studying the CP–violation effects on the neutral and charged Higgs–boson decays, it is worthwhile to examine the present experimental constraints on the above parameter values (11). The charginos, neutralinos, and squarks of the first two generations are sufficiently heavy for the parameter set (11). The lightest squark is the lighter top squark $\tilde{t}_1$, the mass of which can be as low as 200 GeV for $\tan\beta = 4$ and $\Phi = 0^\circ$. The CP–violating phase could weaken the LEP lower limit on the lightest Higgs boson mass significantly [17,18]. In our analysis we show our results when the lightest Higgs–boson mass is above 70 GeV.

The one–loop effective couplings of the CP–odd components of the Higgs boson to the gauge bosons give rise to the electron and neutron EDM's [19] at two–loop level. The effects could be significant for large $\tan\beta$ at large $|A_t|, |A_b|$ and $|\mu|$ so that some region of our parameter space is already excluded by the present 2σ upper bounds on the electron and neutron EDM's : $|d_e| < 0.5 \times 10^{-26}\,e\,\mathrm{cm}$ and $|d_n| < 1.12 \times 10^{-25}\,e\,\mathrm{cm}$, respectively [20]. The dark–shaded region in Fig. 1 is excluded by the electron EDM constraint in the $(\Phi,\ m_A)$ plane for $\tan\beta = 4$ and 10. The unshaded regions give $m_{H_1} < 70$ GeV. The EDM constraints are avoided only :n the lightly shaded region. The excluded region becomes larger for larger $\tan\beta$. Even for $\tan\beta = 4$ as shown in Fig. 1, some parameter space with small m_A and large CP–violating phase can be excluded by the EDM constraint.

It should be noted, however, that the strong two–loop EDM constraints due to the one–loop effective couplings of the CP–odd components of the Higgs bosons to the gauge bosons may not be valid if there appears cancellation among different EDM contributions. Such cancellations may occur between one– and two–loop contributions, or among two–loop contributions themselves [18,21]. In fact, the excluded regions of Fig. 1 disappear if such cancellation takes place at 50 % (20 %) level for $\tan\beta = 4$ ($\tan\beta = 10$). In this paper, we show our results for the whole parameter space (11).

III. NEUTRAL HIGGS BOSON DECAYS

In this section we discuss the phenomenological consequences of the CP–violating Higgs–boson mixing on the total decay widths and decay branching fractions of the neutral Higgs bosons. We choose $\tan\beta = 4$ throughout this section, which leads larger CP–violating effects than the $\tan\beta = 10$ case [7].

The most important decay channels of the Higgs bosons are two–body decays into the heaviest fermions and bosons because the Higgs couplings are proportional to the particle masses. We refer to Ref. [7] for explicit forms of the two–body decay widths of each Higgs boson and the relevant interaction Lagrangians.

The Higgs boson decays into virtual gauge bosons $V^{(*)}V^{(*)}$ are also important for the Higgs bosons of the intermediate mass region, 110 GeV $\lesssim m_H \lesssim 150$ GeV [22–24]. Because of the importance of these 3–body and 4–body decay modes, we extend the previous work [7] to include the Higgs–boson decays into virtual gauge boson pairs and those into a lighter Higgs boson and a virtual Z boson. The partial decay width of $H_i \to V^{(*)}V^{(*)}$ is given by

$$\Gamma(H_i \to V^{(*)}V^{(*)}) = \frac{G_F m_{H_i}^3 \delta_V (\cos\beta O_{2i} + \sin\beta O_{3i})^2}{16\sqrt{2}\pi}$$

$$\times \int_0^{\omega_i} \mathrm{d}x \int_0^{(\sqrt{\omega_i}-\sqrt{x})^2} \mathrm{d}y \frac{\epsilon_V^2 \lambda^{1/2}(\omega_i, x, y)(\lambda(\omega_i, x, y) + 12xy)}{\omega_i^3 \pi^2 [(x-1)^2 + \epsilon_V^2][(y-1)^2 + \epsilon_V^2]}, \tag{13}$$

where $\epsilon_V = \Gamma_V/m_V$, $\omega_i = m_{H_i}^2/m_V^2$, $\delta_W = 2$, $\delta_Z = 1$, and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. For $m_{H_i} > 2m_V$, $1/[(x-1)^2 + \epsilon^2]$ can be approximated by $\pi/\epsilon\,\delta(x-1)$ and the expression of Ref. [7] is recovered for small ϵ. On the other hand, the partial decay width of $H_i \to H_j Z^{(*)}$ is given by

$$\Gamma(H_i \to H_j Z^{(*)}) = \frac{G_F m_{H_i}^3 g_{H_i H_j Z}^2}{8\sqrt{2}\pi} \int_0^{(\sqrt{\omega_i}-\sqrt{\omega_j})^2} \mathrm{d}x \frac{\epsilon_Z \lambda^{3/2}(\omega_i, \omega_j, x)}{\omega_i^3 \pi [(x-1)^2 + \epsilon_Z^2]}, \tag{14}$$

where $g_{H_i H_j Z} = O_{1i}(\cos\beta\, O_{3j} - \sin\beta\, O_{2j}) - O_{1j}(\cos\beta\, O_{3i} - \sin\beta\, O_{2i})$.

For the parameter set (11), the decays of two heavy Higgs bosons into sfermion pairs are kinematically allowed and their decay widths are given by

$$\Gamma(H_i \to \tilde{f}_j^* \tilde{f}_k) = \frac{N_C}{16\pi m_{H_i}} |g_{\tilde{f}_j \tilde{f}_k}^i|^2 \lambda^{1/2}(1, m_{\tilde{f}_j}^2/m_{H_i}^2, m_{\tilde{f}_k}^2/m_{H_i}^2), \tag{15}$$

where $N_C = 3(1)$ for squarks (sleptons). Finally, the Higgs boson decay width into a gluon pair including the sfermion contribution to the decay width is given by

$$\Gamma(H_i \to g\,g) = \alpha_s^2 \frac{m_{H_i}}{32\pi^3} \left(|S_i^g(m_{H_i}^2)|^2 + |P_i^g(m_{H_i}^2)|^2 \right), \tag{16}$$

where the dimensionless form factors $S_i^g(s)$ and $P_i^g(s)$ are given by

$$S_i^g(s) = \sum_{f=t,b} \left\{ g_{sff}^i \frac{\sqrt{s}}{m_f} F_{sf}\left(\frac{s}{4m_f^2}\right) + \frac{1}{4} \sum_{j=1,2} g_{\tilde{f}_j \tilde{f}_j}^i \frac{\sqrt{s}}{m_{\tilde{f}_j}^2} F_0\left(\frac{s}{4m_{\tilde{f}}^2}\right) \right\},$$

$$P_i^g(s) = \sum_{f=t,b} g_{pff}^i \frac{\sqrt{s}}{m_f} F_{pf}\left(\frac{s}{4m_f^2}\right). \tag{17}$$

The couplings g_{sff}^i, g_{pff}^i, and $g_{\tilde{f}_j \tilde{f}_j}^i$ and the functions F_{sf}, F_{pf}, and F_0 are given, for example, in [9].

Figure 2 shows the dominant partial branching fractions of the lightest Higgs boson decays with respect to the mass m_{H_1} for five different values of the CP phase Φ : $\Phi = 180^\circ$, 150°, 120°, 90°, and 60°. The lightest Higgs boson mass is found to be less than 120 GeV and, as a result, the main decay channels are $b\bar{b}$ (upper solid line), $\tau^+\tau^-$ (dashed line), $c\bar{c}$ (dotted line), and gg (lower solid line). We note that the pattern of the branching fractions of the main decay channels, $H_1 \to b\bar{b}$ and $\tau^+\tau^-$, is almost independent of the CP phase. However, for larger m_{H_1}, the branching fraction of the decay $H_1 \to W^{(*)}W^{(*)}$ becomes comparable to that of the two–gluon channel for $\Phi = 120^\circ$ and it can be as large as that of the

$\tau^+\tau^-$ channel for $\Phi = 90^{\rm o}$ [2].

On the other hand, the partial branching fractions for the $H_{2,3}$ decays are very sensitive to the CP phase Φ and strongly dependent on their masses $m_{H_{2,3}}$ as clearly shown in Figs. 3 and 4. The upper solid line in each frame is for $H_{2,3} \to b\bar{b}$ and the lower solid line for $H_{2,3} \to g\,g$. The dashed line is for $H_{2,3} \to \tau^+\tau^-$, the dotted-line for $H_{2,3} \to c\bar{c}$, and two dash–dotted lines are for the channels $H_2 \to W^{(*)}W^{(*)}$ (upper line) and $Z^{(*)}Z^{(*)}$ (lower line), respectively. In addition, the thick solid line shows the branching fractions for $H_{2,3} \to t\bar{t}$, the thick dashed line is for $H_{2,3} \to H_1 Z^{(*)}$, and the thick dotted-line is for $H_{2,3} \to H_1 H_1$. Finally, the thick dash–dotted lines are for the $\tilde{t}_i\overline{\tilde{t}_j}$ decay channels. The decay channels forbidden in the CP invariant theories are marked by filled stars in the legend. These channels become significant as the phase Φ differs from $180^{\rm o}$, being the most dominant channels of the heavy Higgs bosons for non–trivial values of Φ.

Relegating the detailed descriptions of the 2–body decay channels of the heavy Higgs bosons to Ref. [7], it is worthwhile to summarize a few interesting features of the decay modes into virtual vector boson pairs and those into top–squark pairs :

- The decays $H_2 \to W^{(*)}W^{(*)}$ could become the second dominant channel for $M_{H_2} \le 2\,M_W$ for a large range of nontrivial Φ. And the branching fraction of the same decay channel for the heaviest Higgs boson, $H_3 \to W^{(*)}W^{(*)}$, is also sizable for the similar range of Φ.

- The decay channel $H_2 \to \tilde{t}_1\overline{\tilde{t}_1}$, forbidden in the CP invariant theories, overwhelms all the other decay channels for the most range of Φ, once the mode is kinematically allowed. On the contrary, the branching fraction of the decay channel $H_3 \to \tilde{t}_1\overline{\tilde{t}_1}$ decreases as Φ differs from $180^{\rm o}$, exhibiting a typical two–state mixing between two Heavy Higgs bosons for large m_A.

- For the parameter set (11), the non–diagonal decay channels into $H_3 \to \tilde{t}_1\overline{\tilde{t}_2}$ and $\tilde{t}_2\overline{\tilde{t}_1}$ dominate almost all the other decay channels of H_3 once the channel is kinematically allowed when Φ differs significantly from $180^{\rm o}$. The dominance of these channels can be clearly seen near the right end of each frame in Fig. 4 with $\Phi = 90^{\rm o}$ and $60^{\rm o}$.

The dominance of the top–squark channels in the heavy Higgs-boson decays is mainly due to the large values of $|A_t|$ and $|\mu|$ in Eq. (11), which enhance the Higgs–boson couplings to the top squarks. For the particular parameter set (11), two top squarks mix with each other (almost) maximally, leading to a significant suppression of the coupling of CP–even Higgs bosons to non–diagonal top squarks.

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[2]It is worthwhile to note that the decay mode $H_1 \to \gamma\gamma$ with $m_{H_1} \le 130$ GeV is crucial for the detection of the lightest Higgs boson at the CERN Large Hadron Collider (LHC) despite its very low branching fraction. A detailed analysis of the dependence of this important decay mode on the CP phases is to be reported elsewhere [25].

IV. CHARGED HIGGS BOSON DECAYS

Below the $H^+ \to t\bar{b}$ decay threshold, the decay channel $H^+ \to W^{+(*)}H_1$, involving the lightest Higgs boson, constitutes one of the major decay channels of the charged Higgs boson together with the decay channel $H^+ \to \tau^+\nu_\tau$. The fermionic decay channels $H^+ \to t\bar{b}$ and $\tau^+\nu_\tau$ are not affected by the CP–violating neutral Higgs mixing. On the contrary, the coupling of the $H^\pm W^\mp H_1$ vertices depend strongly on the mixing among neutral Higgs bosons such that the phase Φ can affect the branching fractions of the charged–Higgs–boson decays.

The interaction of the charged Higgs boson with a neutral Higgs boson and a W boson in the presence of the CP–violating neutral–Higgs–boson mixing is described by the Lagrangian

$$\mathcal{L}_{H^\pm W^\mp H_i} = \frac{ig}{2}\left(\sin\beta\, O_{2i} - \cos\beta\, O_{3i} + i\, O_{1i}\right)\left(H_i\, \overset{\leftrightarrow}{\partial^\mu}\, H^+\right)W_\mu^- + \text{H.c.}\,, \tag{18}$$

in contrast to which the couplings of the neutral Higgs fields to vector bosons are determined by the Lagrangian

$$\mathcal{L}_{H_i VV} = gm_W(\cos\beta O_{2i} + \sin\beta O_{3i})\, H_i\left[W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2}Z_\mu Z^\mu\right]. \tag{19}$$

Note that if the $H_i VV$ couplings are suppressed, the $H^\pm W^\mp H_i$ couplings are enhanced and vice versa because of the orthogonality of the mixing matrix O: $O_{1i}^2 + O_{2i}^2 + O_{3i}^2 = 1$. It was recently shown [3,17] that the experimental lower limits from the processes $e^+e^- \to ZH_1$ and $e^+e^- \to \nu\bar{\nu}H_1$ on m_{H_1} and $\tan\beta$ could be significantly relaxed with the suppression of the $H_1 VV$ couplings due to the CP–violating neutral Higgs–boson mixing. In this case, the decay channel $H^\pm \to W^{\pm(*)}H_1$ plays a crucial role in confirming the existence of the CP–violating neutral–Higgs–boson mixing because this mode must be enhanced due to the orthogonality relation.

The explicit forms of the two– and three–body decay widths of the fermionic modes, $H^+ \to t^{(*)}\bar{b}, \tau^+\nu_\tau$, can be found in Ref. [23]. So, we present in the present work just the expression of the decay width of $H^+ \to W^{+(*)}H_i$:

$$\Gamma(H^+ \to W^{+(*)}H_i) = \frac{g^2 m_{H^\pm}\left|g_{H^\pm W^\mp H_i}\right|^2}{64\pi^2}\int_0^{(1-\kappa_{H_i})^2}\mathrm{d}x\,\frac{\gamma_W/\kappa_W\,\lambda^{3/2}(1,x,\kappa_{H_i}^2)}{(x-\kappa_W^2)^2 + \kappa_W^2\gamma_W^2}\,, \tag{20}$$

where $g_{H^\pm W^\mp H_i} = \sin\beta\, O_{2i} - \cos\beta\, O_{3i} + i\, O_{1i}$, $\kappa_x = m_x/m_{H^\pm}$, and $\gamma_W = \Gamma_W/m_{H^\pm}$. For the charged Higgs–boson mass $m_{H^\pm} \geq m_{H_i} + m_W$, the decay width simplifies in the narrow–width approximation to

$$\Gamma(H^+ \to W^+ H_i) = \frac{G_F m_{H^\pm}^3\left|g_{H^\pm W^\mp H_i}\right|^2}{8\sqrt{2}\pi}\lambda^{3/2}(1,\kappa_W^2,\kappa_{H_i}^2)\,. \tag{21}$$

For our numerical analysis of the charged–Higgs–boson decays, we take $\tan\beta = 4$ and the parameter set (11) as in the analysis for the neutral Higgs–boson decays. Fig. 5 (a)

shows the branching fraction $\mathcal{B}(H^+ \to W^{+(*)}H_1)$ on the plane of $m_{H^\pm}$ and Φ. The allowed region is divided into 4 parts according to the values of the branching fraction; $\mathcal{B} \geq 0.4$ (filled squares), $0.1 \leq \mathcal{B} < 0.4$ (filled circles), $0.04 \leq \mathcal{B} < 0.1$ (filled triangles), and $\mathcal{B} < 0.04$ (dots). The partial branching fraction $\mathcal{B}$ is enhanced with large CP violating phase such that this mode could be significant for larger $m_{H^\pm}$ in the CP non–invariant case in contrast to the CP invariant case ($\Phi = 180^\circ$). The total decay width of the charged Higgs boson is shown in Fig. 5 (b) as a function of $m_{H^\pm}$. Each line corresponds to $\Phi = 180^\circ$(thick line), $\Phi = 80^\circ$(dash–dotted line), $\Phi = 70^\circ$(dotted line), $\Phi = 60^\circ$(dashed line), and $\Phi = 40^\circ$(solid line). In the region with $\Phi \lesssim 80^\circ$ and $m_{H^\pm} \lesssim 300$ GeV, the total decay width is also significantly affected by the CP phase Φ. We show the branching fractions of the charged Higgs boson in Fig. 5 (c) and (d) as a function of $m_{H^\pm}$ for two representative values of Φ: $\Phi = 180^\circ$ and $\Phi = 70^\circ$. The thick solid line is for $H^+ \to t^{(*)}\bar{b}$, the solid line for $H^+ \to \tau^+\nu_\tau$, and the dashed line for $H^+ \to W^{+(*)}H_1$. We find that the decay channel $H^+ \to W^{+(*)}H_1$ is strongly enhanced as the phase Φ differs from 180°, and the channel becomes the most dominant decay mode of the charged Higgs boson for $m_{H^\pm} \lesssim 300$ GeV when the values of Φ are less than 70°.

To summarize, the charged Higgs boson decays into a W boson and a neutral Higgs boson are closely related to the neutral Higgs boson decays into gauge boson pairs due to the orthogonality of the mixing matrix O in Eq. (10). As a result, the neutral and charged Higgs–boson decays, significantly affected by the phase Φ, are complementary in confirming the loop–induced CP violation in the MSSM Higgs sector.

V. CONCLUSIONS

Based on the new calculation of the mass matrix of the neutral Higgs bosons which is valid for any values of the relevant SUSY parameters, we have re–evaluated the decays of the Higgs bosons in the MSSM with radiative Higgs–sector CP violation. The present work extends the previous one [7] by including all the neutral Higgs–boson decay modes containing virtual gauge bosons and those into top–squark pairs, implementing squark–loop contributions to the two–gluon decay channel. We also study the decays of the charged Higgs boson in the presence of the non–trivial CP–violating mixing. In addition, we have discussed possible constraints from the electron EDM measurements on the CP phase Φ.

We have found that the branching fractions of the main decay channels of the lightest Higgs boson, whose mass is less than 120 GeV for $\tan\beta = 4$, are insensitive to the CP phase(s) and the Higgs boson mass. However, the sub–dominant decay channel $H_1 \to V^{(*)}V^{(*)}$, which is sensitive to the CP phase Φ, can become quite significant as the CP phase differs from 180°. On the other hand, the decays of the heavy neutral Higgs bosons, which are almost degenerate for $m_{H^\pm} > 2\,m_Z$, depend very strongly on the phase. Below the thresholds of the decays into top–quark and top-squark pairs, the main decay channels of the heavy Higgs bosons consist of bottom–quark and tau–lepton pairs, (virtual) gauge bosons, Higgs and (virtual) Z bosons, and Higgs bosons. The relative importance of those decay channels depends significantly on the size of the CP–violating mixing. The

decay channels $H_i \to t\bar{t}, \tilde{t}_1\overline{\tilde{t}_1}, \tilde{t}_1\overline{\tilde{t}_2}$ overwhelm all the other decay channels once these channels are kinematically allowed. The relative importance of these decay modes also depends crucially on the the CP phase(s) and the heavy Higgs–boson masses. Finally, the charged–Higgs–boson decays into a (virtual) W boson and a lightest neutral Higgs boson become dominant when the neutral Higgs–boson decays into gauge boson pairs are suppressed and the charged–Higgs–boson mass is less than 300 GeV.

To conclude, the radiative Higgs sector CP violation induced from the third–generation sfermion sectors could alter the patterns of the neutral and charged Higgs–boson decays significantly from those in the CP invariant theories.

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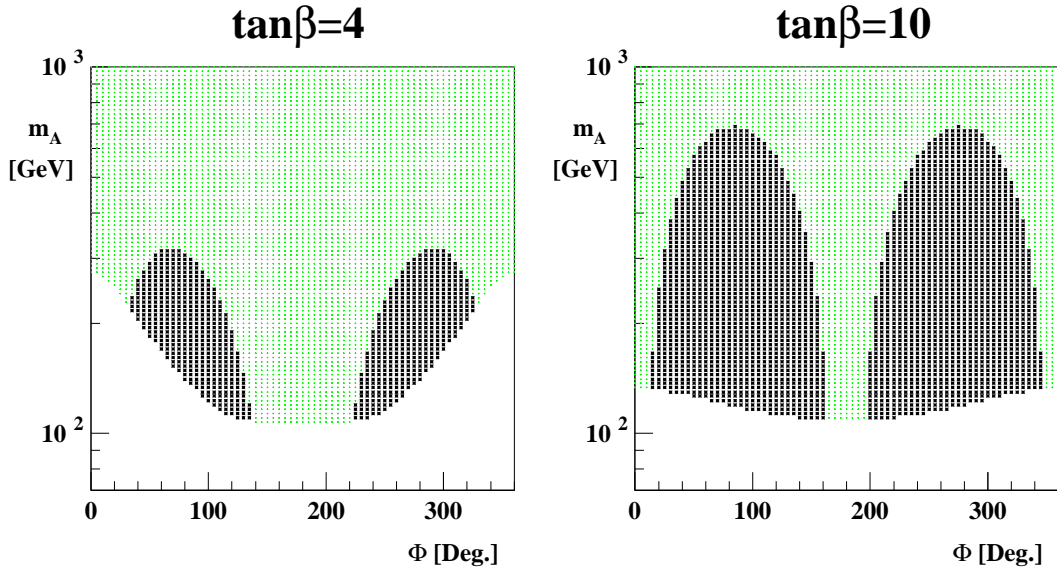FIG. 1. The regions of m_A and Φ allowed by the experimental electron EDM constraint for the parameter set (11) are lightly shaded for both $\tan\beta = 4$ (left) and $\tan\beta = 10$ (right). The dark–shaded regions are excluded by the electron EDM constraint while the unshaded regions give $m_{H_1} < 70$ GeV.

FIG. 2. The partial branching fractions of the lightest Higgs–boson decays with respect to the mass m_{H_1} for $\tan\beta = 4$ and five values values of the CP phase Φ ; $\Phi = 180^o, 150^o, 120^o, 90^o$, and 60°. The upper solid line in each frame is for $H_1 \to b\bar{b}$, the dashed line for $H_1 \to \tau^+\tau^-$, the dotted line for $H_1 \to c\bar{c}$, and lower solid line for $H_1 \to gg$. In addition, the upper dash–dotted line is for $H_1 \to W^{(*)}W^{(*)}$ and the lower one for $H_1 \to Z^{(*)}Z^{(*)}$.
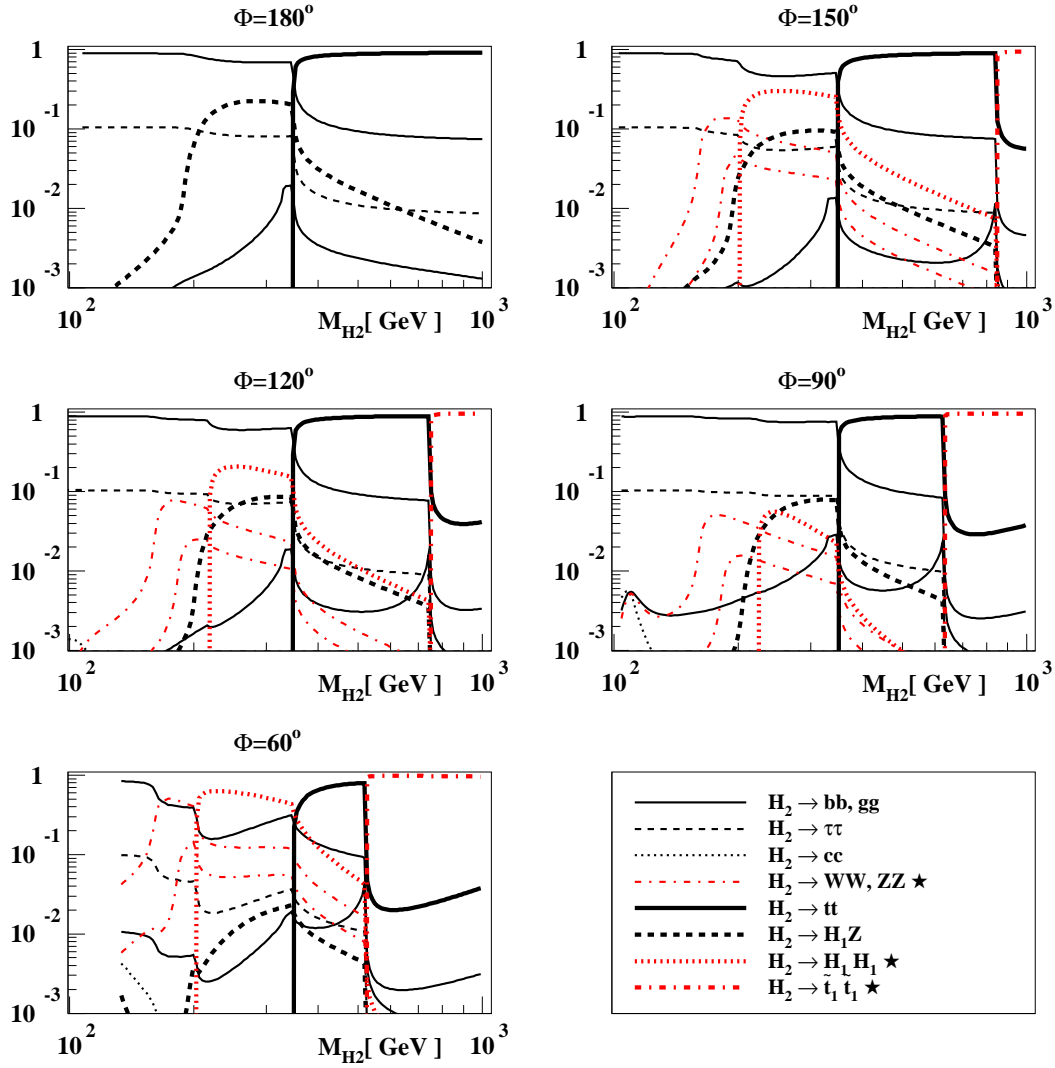
FIG. 3. The partial branching fractions for the H_2 decay channels with respect to the mass m_{H_2} for five values of the CP phase Φ for $\tan\beta = 4$ as in Fig. 2. The decay channels marked by filled stars in the legend are forbidden in the CP invariant theories.
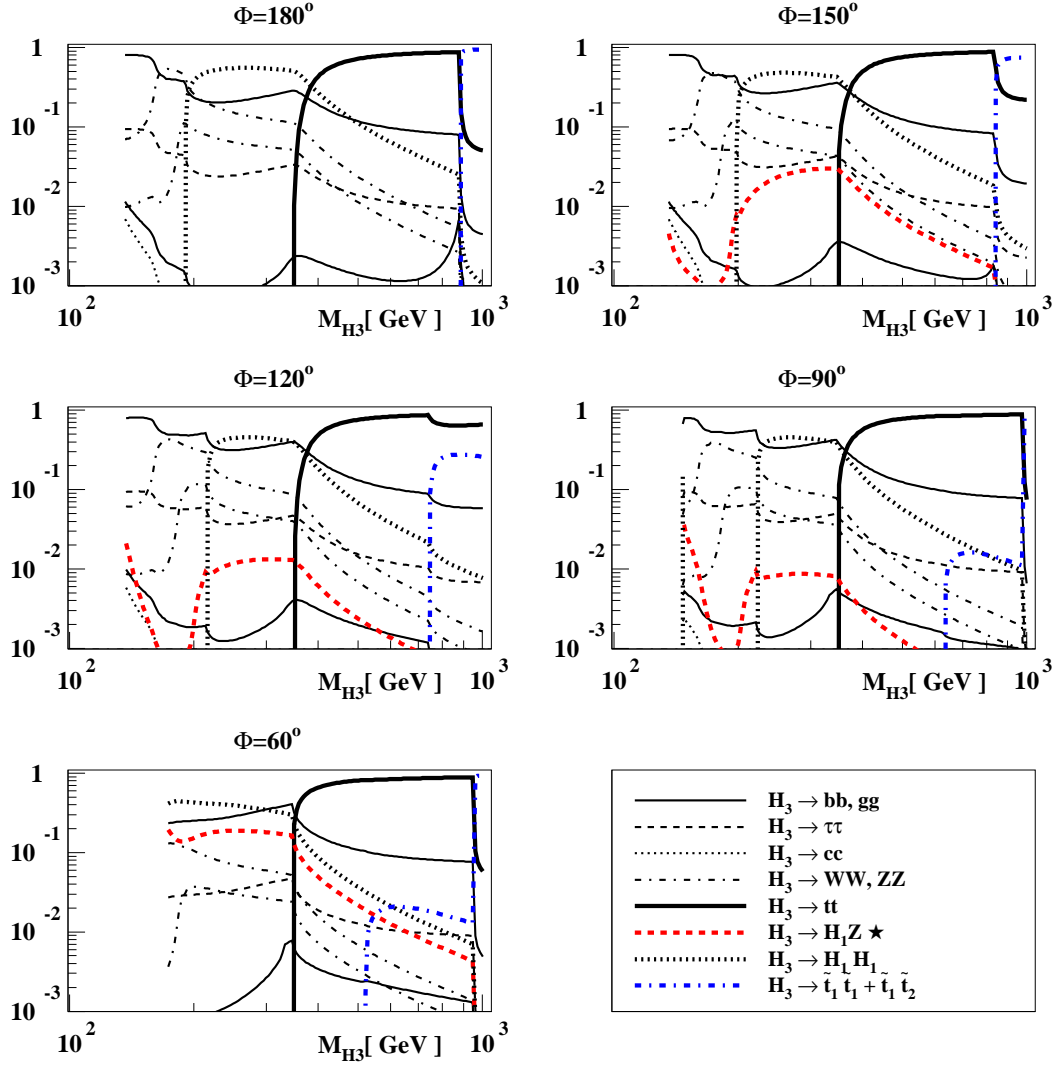
FIG. 4. The partial branching fractions for the H_3 decay channels with respect to the mass m_{H_3} for $\tan\beta = 4$ and five values of the CP phase Φ as in Figs. 2 and 3. The thick dash–dotted line is for the sum of the $H_3 \to \tilde{t}_1\bar{\tilde{t}}_1$ and $H_3 \to \tilde{t}_1\bar{\tilde{t}}_2 + \tilde{t}_2\bar{\tilde{t}}_1$ modes. The decay channel $H_3 \to H_1 Z$ marked by a filled star in the legend is forbidden in the CP invariant theories.
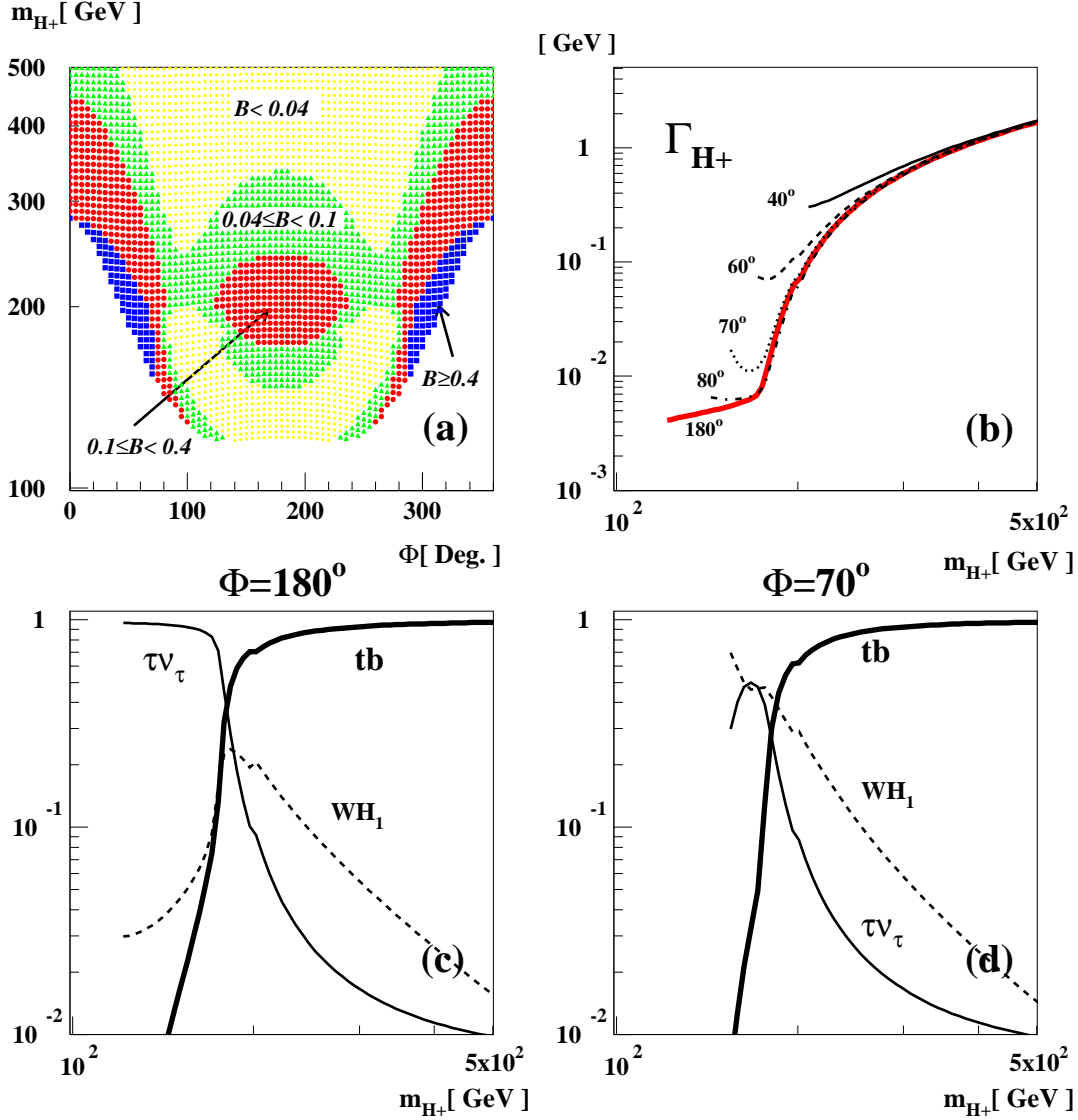
FIG. 5. The branching fractions of the charged Higgs boson and its total decay width for $\tan\beta = 4$ and the parameter set (11); (a) the branching fraction $\mathcal{B}(H^+ \to W^+ H_1)$ on the plane of $m_{H^\pm}$ and Φ, (b) the total decay width Γ_{H^+} as a function of $m_{H^\pm}$ for five values of Φ; $\Phi = 180^\circ$(thick line), $\Phi = 80^\circ$(dash–dotted line), $\Phi = 70^\circ$(dotted line), $\Phi = 60^\circ$(dashed line), and $\Phi = 40^\circ$(solid line), (c) and (d) the branching fractions of the main charged–Higgs–boson decays as a function of $m_{H^\pm}$ for $\Phi = 180^\circ$ and $\Phi = 70^\circ$, respectively.